Geometric and large scale problems in SPDEs

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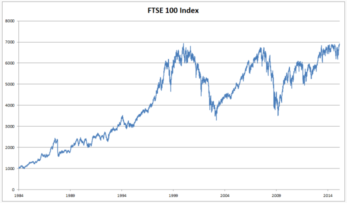
The mathematics of subjective probability
University of Milano-Bicocca

Roughness in random systems

Many random systems are rough.

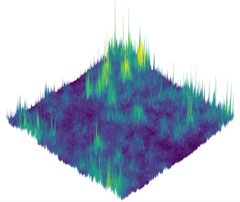
Examples:

Trajectories of stock prices



FTSE 100 Index

Liouville Quantum Gravity



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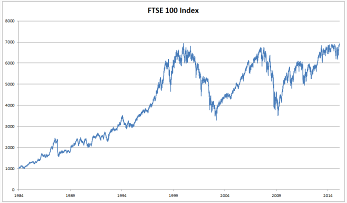

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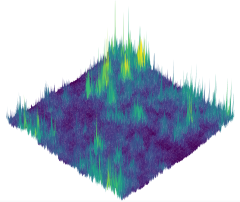

Difficulty: calculus requires regularity of functions.

Goal: explain how to handle roughness.

Stock prices

- **Black–Scholes** (1967), **Merton** (1973) – infinitesimal change in price proportional to current price:

$$S_{t+\delta t} = S_t + \sigma S_t (B_{t+\delta t} - B_t) \ .$$

Brownian noise: $B_{t+\delta t} - B_t$ is
  - independent of past &
  - distributed like normal random variable with variance δt.

- In differential form:

$$\frac{\mathrm{d}S_t}{\mathrm{d}t} = \sigma S_t \frac{\mathrm{d}B_t}{\mathrm{d}t} \ .$$

- How to interpret this equation?

- If B is differentiable, then

$$S_t = S_0 e^{\sigma B_t}.$$

Black–Scholes–Merton model

- What if B_t is Brownian noise?
- Take discrete approximation: for $N > 0$, solve **difference equation**

$$S_{(k+1)/N} = S_{k/N} + \sigma S_{k/N}(B_{(k+1)/N} - B_{k/N}) \; .$$

where $B_0, B_{1/N}, B_{2/N}, \ldots$ is symmetric random walk on $\frac{1}{\sqrt{N}}\mathbb{Z}$.

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- What? Why $\sigma^2 t/2$?
- Can be **guessed**: average of $e^{\sigma B_t}$ is $e^{\sigma^2 t/2}$.
- Brownian motion is **rough**: sensitive to approximation scheme.

Credit: Jacobs, Konrad

K. Itô: rewrite in integral form

$$S_t = S_0 + \int_0^t \sigma S_r \, \mathrm{d}B_r$$

and define the integral $\int \sigma S_r \, \mathrm{d}B_r$ as limit *in probability* of Riemann sums

$$\lim_{n \to \infty} \sum_{[r,s] \in \pi_n} S_r (B_s - B_r) \, .$$

Solve for S in space of *stochastic processes*.

Intrinsically probabilistic: difficult to control the solution map $B \mapsto S$.

Naive attempt

Try to solve SDE $S\colon [0, T] \to \mathbb{R}^n$

$$\frac{\mathrm{d}}{\mathrm{d}t} S_t = f(S) \frac{\mathrm{d}B_t}{\mathrm{d}t}$$

- $\frac{\mathrm{d}B_t}{\mathrm{d}t}$ is white noise,
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- Integration adds one derivative $\Rightarrow S \in \mathcal{C}^{\frac{1}{2}-\kappa} \Rightarrow f(S) \in \mathcal{C}^{\frac{1}{2}-\kappa}$.

Naive attempt

Theorem (Multiplication of distributions)

Suppose $\alpha \le \beta$. Multiplication

$$(f, g) \mapsto fg \ ,$$

defined on

$$\mathcal{C}^\infty \times \mathcal{C}^\infty \to \mathcal{C}^\infty \ ,$$

extends to continuous map

$$\mathcal{C}^\alpha \times \mathcal{C}^\beta \to \mathcal{C}^\alpha$$

if and only if $\alpha + \beta > 0$.

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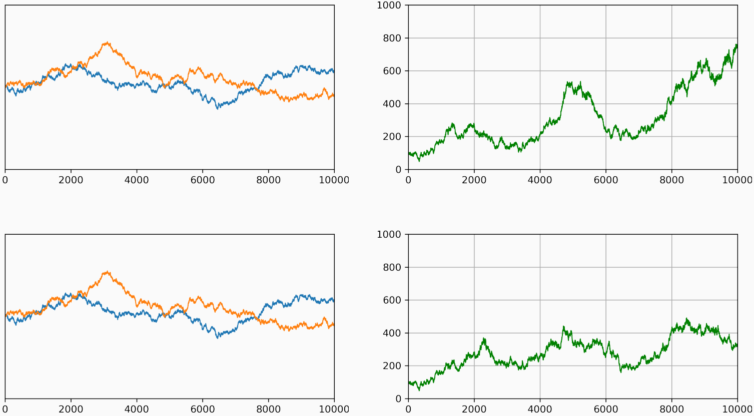
- $-\frac{1}{2} - \kappa + \frac{1}{2} - \kappa < 0 \Rightarrow f(S)\frac{\mathrm{d}B_t}{\mathrm{d}t}$ analytically ill-defined.

Theorem: there exists f such that any Banach space on which the solution map $B \mapsto S$ is continuous cannot contain smooth functions and 2D Brownian motion. [Lyons '91]

Instability of solution map

Equations with two noises: $\mathrm{d}S_t = f_1(S_t)\,\mathrm{d}B_t^{(1)} + f_2(S_t)\,\mathrm{d}B_t^{(2)}$.



Closer look

Consider B smooth $\Rightarrow \mathrm{d}S_t = f(S_t)\,\mathrm{d}B_t$ well-posed.

Expand f in Taylor series

$$f(S_t) = f(S_0) + f'(S_0)(S_t - S_0) + \dots$$

and substitute back into equation:

$$\begin{aligned} S_t &= S_0 + \int_0^t f(S_s)\,\mathrm{d}B_s \\ &= S_0 + f(S_0)\int_0^t \mathrm{d}B_s + f'(S_0)\int_0^t \Big(\int_0^s \mathrm{d}B_r\Big)\,\mathrm{d}B_s + \dots \end{aligned}$$

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If $\int_0^t \left(\int_0^s \mathrm{d}B_r\right)\mathrm{d}B_s$ big (order $\geq t$), cannot ignore second term.

Idea: View solution $\mathrm{d}S_t = f(S_t)\,\mathrm{d}B_t$ as function of $(B_t, \int_0^t \int_0^s \mathrm{d}B_r\,\mathrm{d}B_s)$.

Rough paths

Theorem (Lyons '98)

Fix $\alpha \in (\frac{1}{3}, \frac{1}{2})$. *There exists a metric space of pairs* $(B_t, \int_0^t \int_0^s \mathrm{d}B_r \,\mathrm{d}B_s)$ *with*

$$\sup_{u \neq t} \frac{|B_t - B_u|}{|t-u|^\alpha} + \sup_{u \neq t} \frac{|\int_u^t \int_u^s \mathrm{d}B_r \,\mathrm{d}B_s)|}{|t-u|^{2\alpha}} < \infty$$

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For many stochastic B and approximations $B^{(N)}$

$$\lim_{N \to \infty} \left(B_t^{(N)}, \int_0^t \int_0^s \mathrm{d}B_r^{(N)} \, \mathrm{d}B_s^{(N)} \right) = \left(B_t, \int_0^t \int_0^s \mathrm{d}B_r \, \mathrm{d}B_s \right) . \qquad (\star)$$

($\int_0^t \int_0^s \mathrm{d}B_r \, \mathrm{d}B_s$ **defined** as $\lim_{N \to \infty} \int_0^t \int_0^s \mathrm{d}B_r^{(N)} \, \mathrm{d}B_s^{(N)}$ – Itô vs. Stratonovich)

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The point:

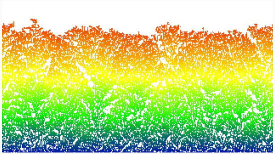
- Probability enters only in showing $(\star)$.
- Alternative approach to SDEs, applies beyond (semi)martingales.

Stochastic (Partial) Differential Equations

Model **randomness**:

$$\partial_t h = \Delta h + F(h, \nabla h, \xi)$$

- stochastic quantisation equations (Yang–Mills, Φ^4_d, Sine–Gordon)
- spread of populations (Parabolic Anderson Model)
- crystal growth (Khardar–Parisi–Zhang: $\partial_t h = \Delta h + (\partial_x h)^2 + \xi$)



Credit: Nils Berglund

Difficulties: solutions h are rough functions or distributions

$\rightsquigarrow$ non-linearity $F(h, \nabla h, \xi)$ ill-defined (e.g. $(\partial_x h)^2$ in KPZ).

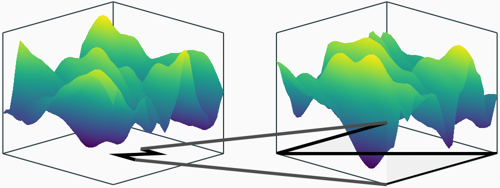
Ultraviolet scaling

Consider cubic equation:

$$(\partial_t - \Delta)u = -u^3 + \xi.$$

Zoom to small scales

$$\tilde{u}(t,x) = \varepsilon^{\beta} u(\varepsilon^2 t, \varepsilon x).$$



$$(\partial_t - \Delta)\tilde{u} = -\varepsilon^{2-2\beta}\tilde{u}^3 + \tilde{\xi}$$

where

$$\tilde{\xi}(t,x) = \varepsilon^{2+\beta}\xi(\varepsilon^2 t, \varepsilon x).$$

For $\xi \in \mathcal{C}^{-2-\beta}$, $\|\tilde{\xi}\|_{\mathcal{C}^{-2-\beta}} \sim \|\xi\|_{\mathcal{C}^{-2-\beta}}$. **Non-linearity disappears** if $\beta < 1$.

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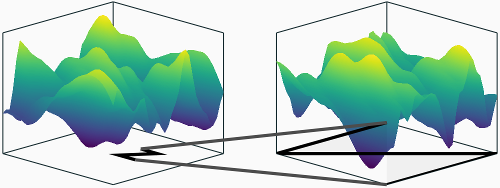

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$\Rightarrow$ on **small scales** u locally looks like solution of $(\partial_t - \Delta)\tilde{u} = \xi$.

E.g. For space-time white noise on $\mathbb{R} \times \mathbb{R}^d$, $\xi \in \mathcal{C}^{>-3} \Leftrightarrow d < 4$.

SPDE - local theory

- regularity structures [Hairer, *Invent. Math.* '14]
- paracontrolled distributions [Gubinelli–Imkeller–Perkowski, *FoM Pi* '15].

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Theorem (Bruned, Chandra, C., Hairer, Zambotti ‘14-‘17)

Sub-critical SPDEs on $\mathbb{R}_+ \times \mathbb{T}^d$ with stationary noise ξ

$$\partial_t A = \Delta A + F(A, \nabla A, \dots)\xi$$

*admit **local** solutions via “renormalised” smooth approximations*

$$\partial_t A^\varepsilon = \mathcal{L} A^\varepsilon + F(A^\varepsilon, \nabla A^\varepsilon, \dots)\xi^\varepsilon + \sum_{i=1}^n C_{i,\varepsilon} F_i(A^\varepsilon, \nabla A^\varepsilon, \dots) \,.$$

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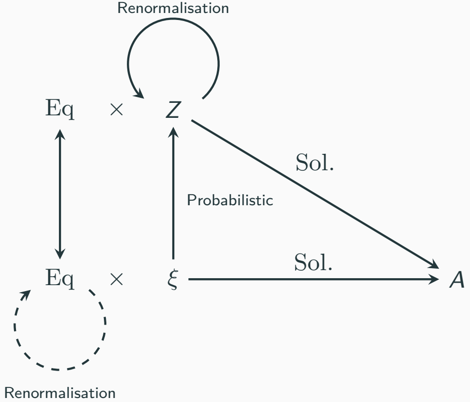
Probabilistic step: build object $Z = \big(\xi^\varepsilon, (G * \xi^\varepsilon)^{:2:} =$ , , $\dots),$
$G = (\partial_t - \mathcal{L})^{-1}$. Renormalisation: $(G * \xi^\varepsilon)^2 \mapsto (G * \xi^\varepsilon)^{:2:} = (G * \xi^\varepsilon)^2 - C^\varepsilon$.

Analytic step: continuous solution map $Z \mapsto A$.

Algebraic step: find counterterms F_i.

Also [Otto, Weber, Sauer, Smith, Linares, Tempelmayr, Tsatsoulis '16-'21]
and renormalisation group: [Kupiainen '16', Duch '21].

Summary



Automatic **local** solution theory.

Global solution theory less developed - only specific equations
[Mourrat–Weber '17, Moinat–Weber '20, Gubinelli–Hofmanova '21, Bringmann–Cao '24,...].

Infrared scaling - global solutions

Consider $(\partial_t - \Delta)u = -u^3 + \xi$ and **critical scaling** $\tilde{u}(t,x) = \varepsilon u(\varepsilon^2 t, \varepsilon x)$.

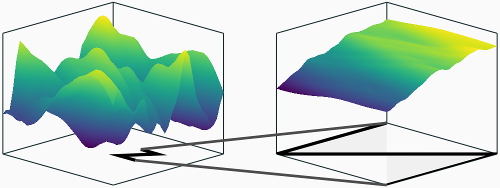
Then

$$(\partial_t - \Delta)\tilde{u} = -\tilde{u}^3 + \tilde{\xi}$$

where $\tilde{\xi}(t,x) = \varepsilon^3 \xi(\varepsilon^2 t, \varepsilon x)$. Then

$$\|\tilde{\xi}\|_{\mathcal{C}^\alpha} \lesssim \varepsilon^{3+\alpha} \|\xi\|_{\mathcal{C}^\alpha}$$

Noise disappears for $\alpha > -3$.



Large scale bound: choose $\varepsilon > 0$ such that $\|\tilde{\xi}\|_{\mathcal{C}^\alpha} \ll 1$ but still order 1.

Then by local stability and coercivity at $\xi = 0$,

$$\|\tilde{u}\|_\infty = \varepsilon \|u\|_{\infty; B_0(\varepsilon)} \lesssim 1 \quad \Rightarrow \quad \|u\|_{\infty; B_0(\varepsilon)} \lesssim \varepsilon^{-1} \lesssim \|\xi\|_{\mathcal{C}^\alpha}^{1/(3+\alpha)} .$$

Coercive equations

Consider a subcritical SPDE

$$(\partial_t - \Delta)u = P(u, \nabla u) + f(u, \nabla u)\xi \; . \qquad (\star)$$

We say $(\star)$ is **coercive** at $\xi \in \mathcal{C}^\alpha$ if, for any solution in the ball $B_0(1)$,

$$|u(z)| \lesssim \|\xi\|_{\mathcal{C}^\alpha}^{\varrho} + |\!|\!| z |\!|\!|^{-\beta}$$

where $|\!|\!| z |\!|\!|$ is distance of z to boundary of $B_0(1)$.

Theorem (C.–Gubinelli '25+)

*Suppose $(\star)$ is **coercive** at $\xi = 0$ and f does not grow too quickly at ∞. Then $(\star)$ is **coercive** for all ξ for a suitable ϱ.*

Space-time localisation. Extends and simplifies [Moinat–Weber '20, Chandra–Moinat–Weber '22, Bonnefoi–CMW '22, Jin–Perkowski '25]

Idea of proof

- Take point z that maximises

$$C := \frac{|u(z)|}{\|\xi\|_{\mathcal{C}^\alpha}^{\varrho} + \|z\|^{-\beta}}$$

- Zoom with infrared scaling around z, so $\tilde{u}(t,x) = \varepsilon^{\beta} u(z + (\varepsilon^2 t, \varepsilon x))$ solves

$$(\partial - \Delta)\tilde{u} = P(\tilde{u}, \nabla \tilde{u}) + \tilde{f}(\tilde{u}, \nabla \tilde{u})\tilde{\xi}\ ,$$

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- Choose ε such that $|\tilde{u}(0)| = 1$. Subcriticality $\Rightarrow$ **if** $C \gg 1$, then $\|\tilde{f}\tilde{\xi}\| \ll 1$.

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- Choose ε such that $|\tilde{u}(0)| = 1$. Subcriticality $\Rightarrow$ **if** $C \gg 1$, then $\|\tilde{f}\tilde{\xi}\| \ll 1$.
- Maximisation of z: $\|\tilde{u}\|_{\infty;B_0(1)} \leq 1 + \delta$.
- **If** $C \gg 1$, coercivity and stability of equation at $\tilde{\xi} = 0 \Rightarrow |\tilde{u}(0)| < 1 - \delta$, which contradicts scaling choice.

Summary

Rough analysis: deterministic approach to stochastic equations.

- Keep track of underlying process **and** additional data.
- Calculus for rough objects.
- Scaling: guide for local and global solution theories.

Thank you for your attention!