The Critical 2d SHF

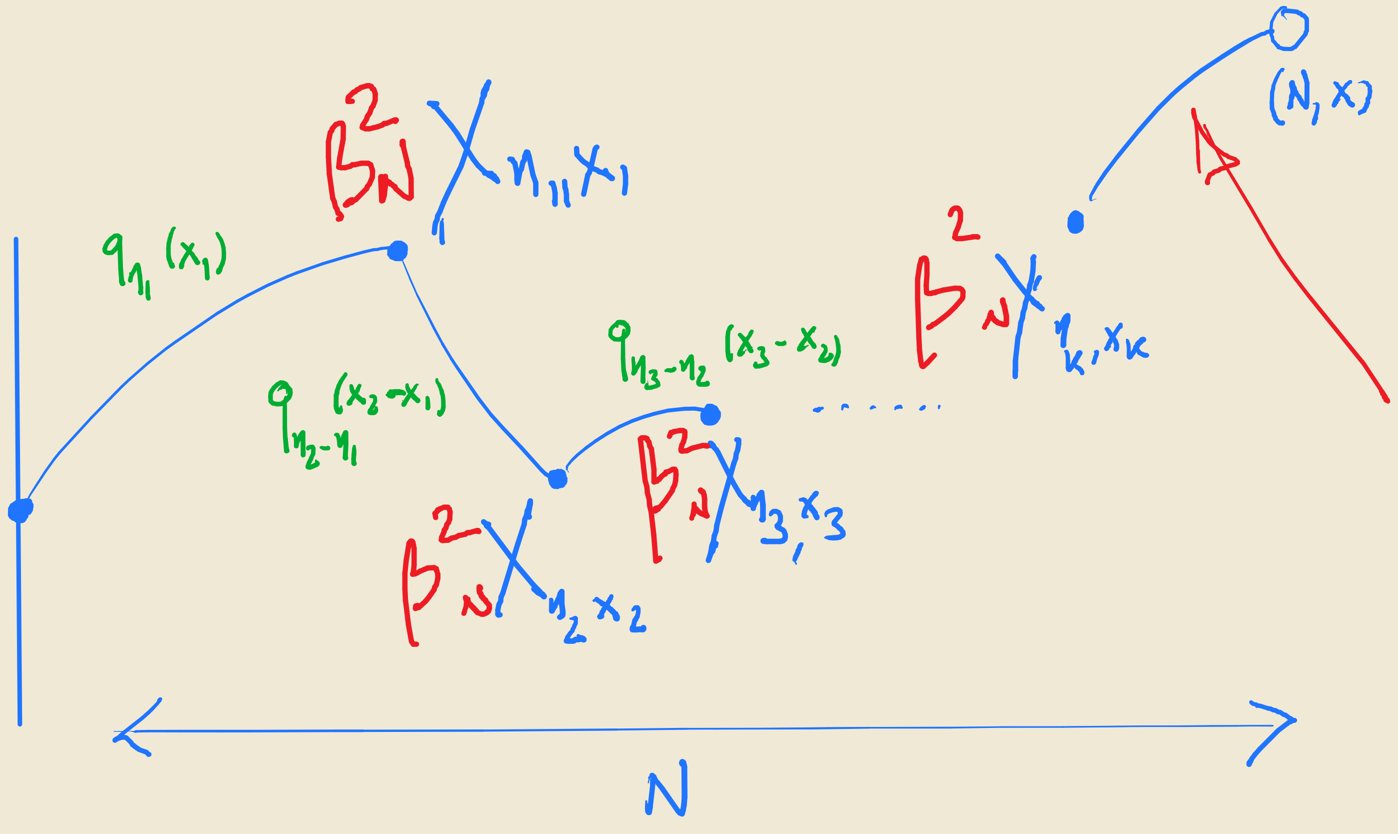
Part III:

tools & problems

Reminder of coarse-graining

MICROSCOPIC MODEL

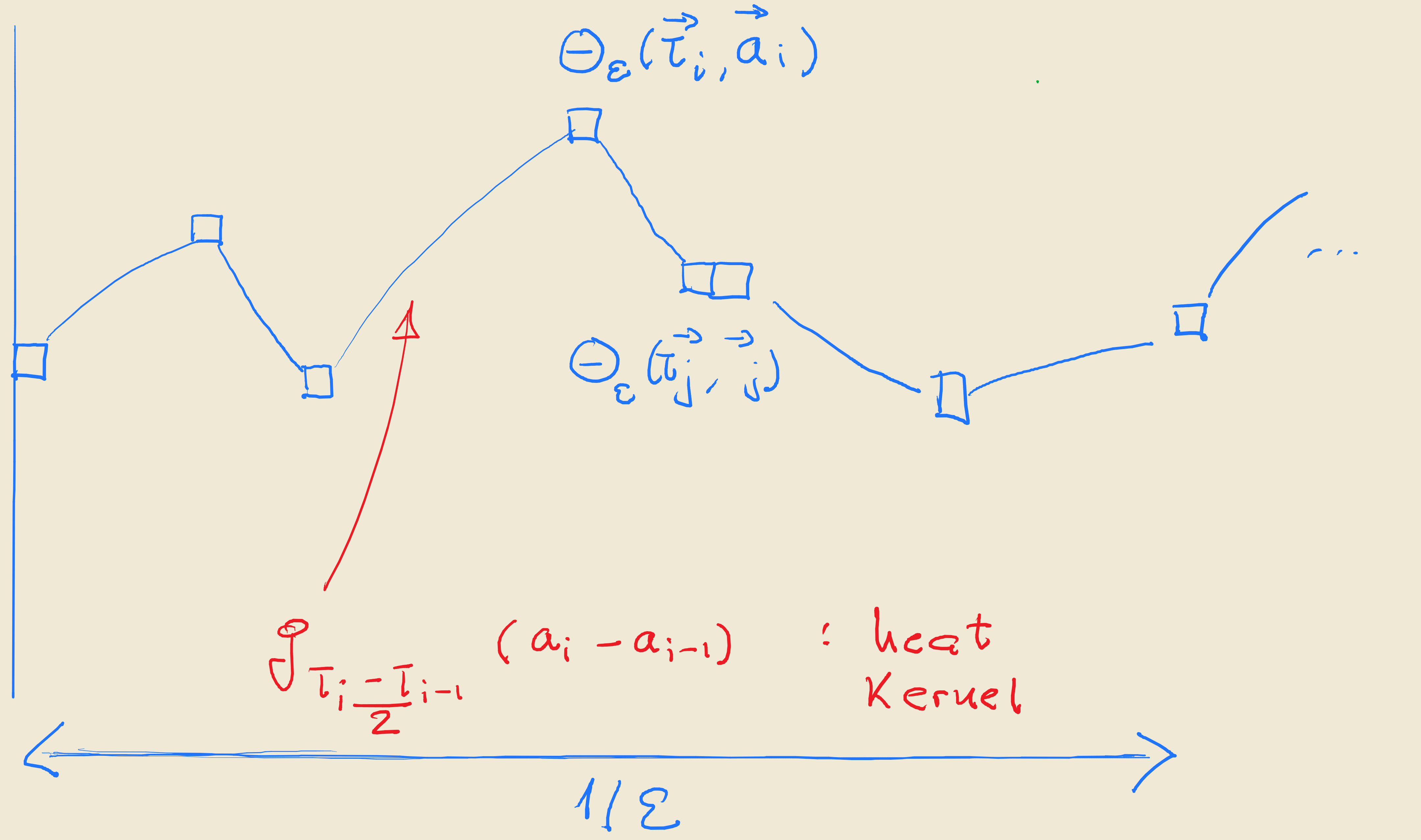
$$\sum_{k} \sum_{\substack{n_1 < \cdots < n_k \\ x_1, \ldots, x_k}}$$



$q_{n_i - n_{i-1}}(x_i - x_{i-1})$: random walk transition

COARSE-GRAIN / MESOSCOPIC MODEL

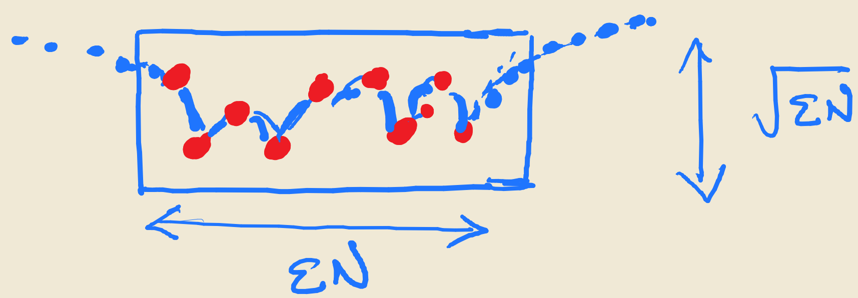
$$\sum_{k} \sum_{\substack{\vec{\tau}_1 \cdots \vec{\tau}_k \in \mathbb{N}^2 \\ \vec{a}_1, \ldots, \vec{a}_k \in (\mathbb{Z}^2)^2}}$$



$g_{\frac{\tau_i - \tau_{i-1}}{2}}(a_i - a_{i-1})$: heat kernel

Coarse Graining maintains critical scaling

$$\mathrm{Var}\left(\frac{1}{N^2} \right.$$



$$\left. \vphantom{\frac{1}{N^2}} \right) \underset{N\to\infty}{\cong} \frac{\sqrt{\pi}}{\log 1/\varepsilon}$$

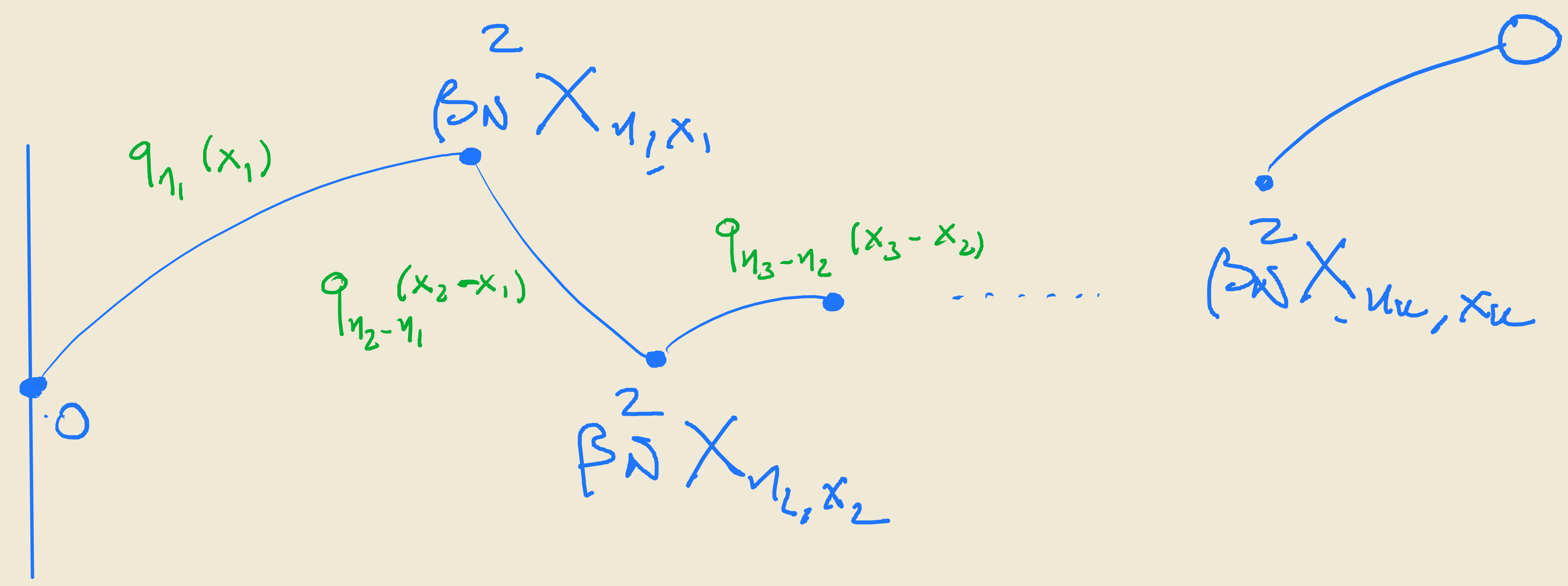
$\Theta_\varepsilon(i,a)$

Variance Computations & the Dickman function

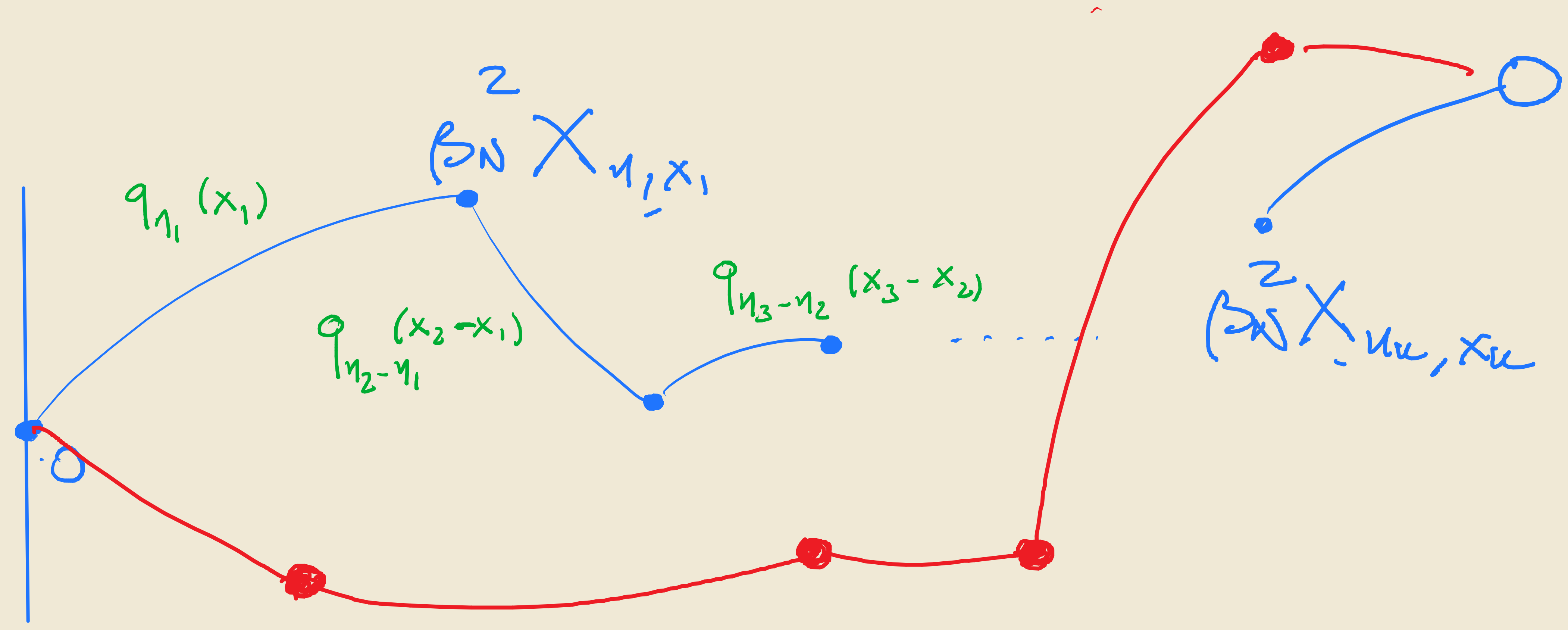
Recall $\beta_N^2 X_{n,y} := e^{\beta_N X_{n,y} - \lambda(\beta_N)} - 1$

$$q_n(x) = \mathbb{P}_0(S_n = x)$$

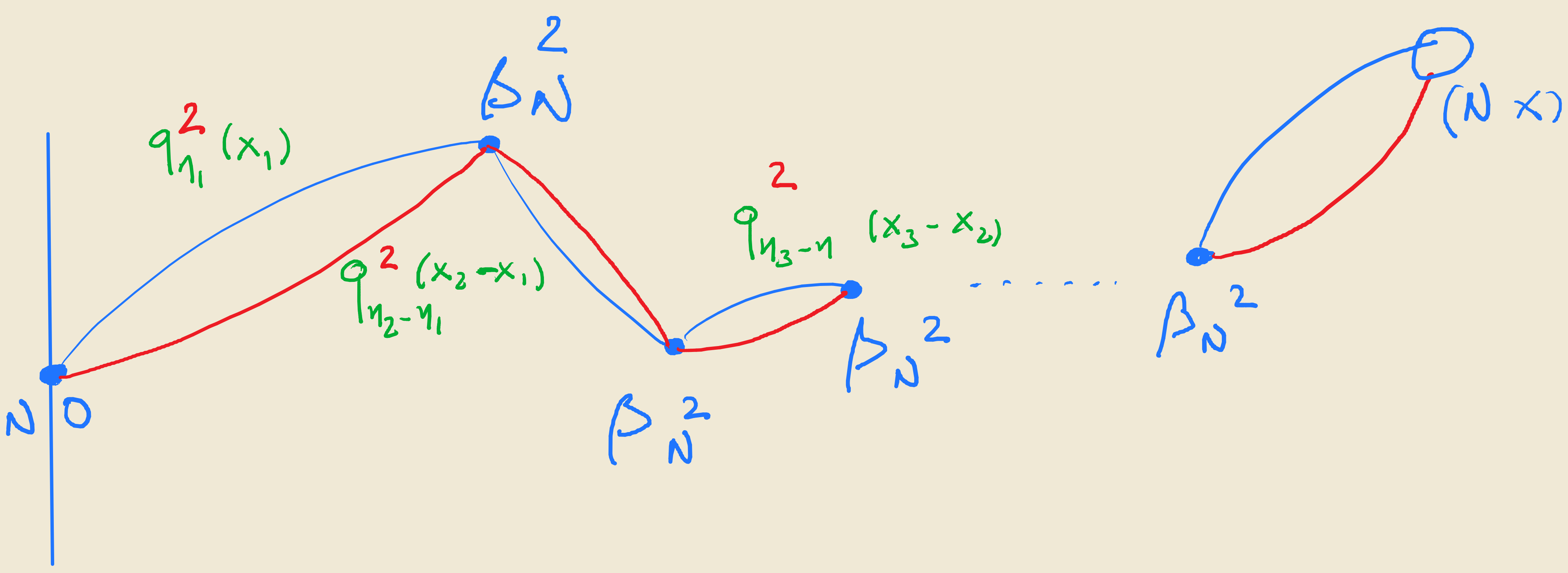
$$Z_N(Nt, \sqrt{N}x) = \sum_k \sum_{\substack{n_1 < \dots < n_k \\ x_1, \dots, x_k}}$$



$$\mathrm{Var}\left(Z_N(Nt, \sqrt{N}x)\right) = \mathbb{E} \sum$$



$$= \sum_k \sum_{\substack{n_1 < \dots < n_k < N \\ x_1, \dots, x_k}}$$

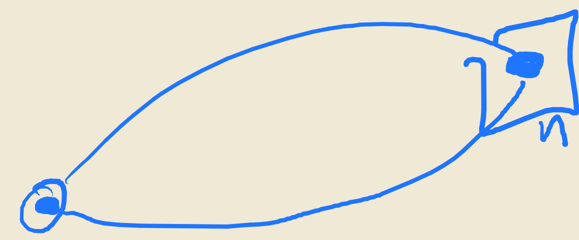


Some computations & asymptotics

$$\mathrm{Var}\left(Z^{\text{crit.}}_{N,\beta_N} \right) = \sum_{k\geq 1} \beta_N^{2k} \sum_{\substack{n_1 < \dots < n_k < N \\ x_1, \dots, x_k}} \prod_{i=1}^{k} q^2_{n_i - n_{i-1}}(x_i - x_{i-1})$$

Sum out space

$$\sum_{x_k} q^2_{n_k - n_{k-1}}(x_k - x_{k-1}) = \sum_{x_k}$$



$$= q_{2(n_k - n_{k-1})}(0) \simeq \frac{1}{\pi} \frac{1}{n_k - n_{k-1}}$$

neglecting periodicity et al. constants

Do the same for $x_{k-1}, x_{k-2}, \dots$ (& neglecting π & other constants ...)

$$\mathrm{Var}\left(Z^{\text{crit}}_{N,\beta_N} \right) = \sum_{k\geq 1} \frac{\beta_N^{2k}}{\pi^k} \sum_{n_1 < \dots < n_k < N} \frac{1}{n_1 (n_2 - n_1) \cdots (n_k - n_{k-1})}$$

$$\mathbb{V}\mathrm{ar}\left(Z^{\mathrm{crit}}_{N,\beta_N}\right) = \sum_{k\geq 1}\left(\frac{\beta_N^2}{\pi}\right)^k \sum_{n_1<\cdots<n_k\leq N} \frac{1}{n_1(n_2-n_1)\cdots(n_n-n_{k-1})}$$

Recall : $\beta_N^2 = \frac{\pi}{\log N}\left(1+\frac{\theta}{\log N}\right)$

and define $T_1, T_2, \ldots$ i.i.d. with

$$\mathbb{P}(T=n) = \frac{1}{\log N}\frac{\mathbb{1}_{n\leq N}}{n} \quad \text{for } n=1,\ldots,N$$

then

$$\mathbb{V}\mathrm{ar}\left(Z^{\mathrm{crit}}_{N,\beta_N}\right) = \sum_{k\geq 1} \frac{\beta_N^{2k}}{\pi^k} \sum_{n_1<\cdots<n_k<N} \frac{1}{n_1(n_2-n_1)\cdots(n_k-n_{k-1})}$$

$$= \sum_{k\geq 1} \frac{1}{(\log N)^k}\left(1+\frac{\theta}{\log N}\right)^k \sum_{n_1<\cdots<n_k<N} \frac{1}{n_1(n_2-n_1)\cdots(n_k-n_{k-1})}$$

$$= \sum_{k\geq 1}\left(1+\frac{\theta}{\log N}\right)^k \mathbb{P}\left(T_1+\cdots+T_k\leq N\right)$$

$$\cong \sum_{k\geq 1} e^{\theta k/\log N}\,\mathbb{P}\left(T_1+\cdots+T_k\leq N\right)$$

$$= \log N\cdot\frac{1}{\log N}\sum_{k\geq 1} e^{\theta k/\log N}\,\mathbb{P}\left(T_1+\cdots+T_{\frac{k}{\log N}\cdot\log N}\leq N\right)$$

$$\approx \log N\int_0^\infty e^{\theta s}\,\underbrace{\mathbb{P}\left(T_1+\cdots+T_{s\log N}\leq N\right)}\,ds$$

$$\downarrow$$

$$\approx \log N\int_0^\infty e^{\theta s}\,\mathbb{P}\left(Y_s\leq 1\right)\,ds$$

Prop $T_1, T_2, \dots$ i.i.d. with $\mathbb{P}(T_i = n) = \frac{1}{\log N} \frac{\mathbb{1}_{n \le N}}{n}$

then

$$\frac{T_1 + \dots + T_{s \log N}}{N} \longrightarrow Y_s$$

with $(Y_s)_{s>0}$ called the Dickman subordinator, i.e.

Lévy process with jump measure $\nu(dx) = \frac{\mathbb{1}_{x \in (0,1)}}{x} dx$

Proof $\mathbb{E} \exp\left(\frac{\lambda}{N} (T_1 + \dots + T_{s \log N})\right) = \mathbb{E}\left[e^{\lambda/N \, T}\right]^{s \log N}$

$$= \left\{ 1 + \frac{1}{\log N} \sum_{n=1}^{N} \frac{e^{\lambda/N \, n} - 1}{n} \right\}^{s \log N}$$

$$\approx \exp\left\{ s \sum_{n=1}^{N} \frac{e^{\lambda/N \, n} - 1}{n} \right\}$$

$$\approx \exp\left\{ s \int_0^1 (e^{\lambda x} - 1) \frac{dx}{x} \right\}$$

Refined variance asymptotics & local limit theorems

$$\mathbb{V}\mathrm{ar}\left(Z^{\mathrm{crit.}}_{N,\beta_N}(0,0;\, n,x) \right) \sim \frac{(\log N)^2}{\pi N^2}\, G_\theta\!\left(\frac{n}{N}\right) g_{\frac{n}{4N}}\!\left(\frac{x}{\sqrt{N}}\right)$$

where

$$G_\theta(t) = \int_0^\infty e^{\theta s}\, \mathrm{P}(Y_s = t)\, ds$$

$$\sim \frac{1}{t(\log t)^2} \left\{ 1 + \frac{\theta + o(1)}{\log 1/t} \right\}$$

On the Dickman function

Dickman subordinator: Lévy measure $\frac{\mathbb{1}_{(0,1)}}{x}\,dx$

Dickman function:

$$f_s(t) = \begin{cases} \dfrac{s\,t^{s-1}}{\Gamma(s+1)}\, e^{-\gamma s} & , \ t \in (0,1) \\[2ex] \dfrac{s\,t^{s-1} e^{-\gamma s}}{\Gamma(s+1)} - s\,t^{s-1} \displaystyle\int_0^{t-1} \frac{f_s(a)}{(1+a)^s}\, ds & , \ t > 1 \end{cases}$$

<u>Number theory</u>

Tenenbaum

"Analytic & Probabilistic Number Th."

$$\rho(1/t) := e^{\gamma} f_1(1/t) =$$

$$\simeq \text{Prob}\{\text{largest prime factor from } [1,\dots,N] \le N^t\}$$

<u>Random permutations</u>

Arratia - Barbour - Tavare

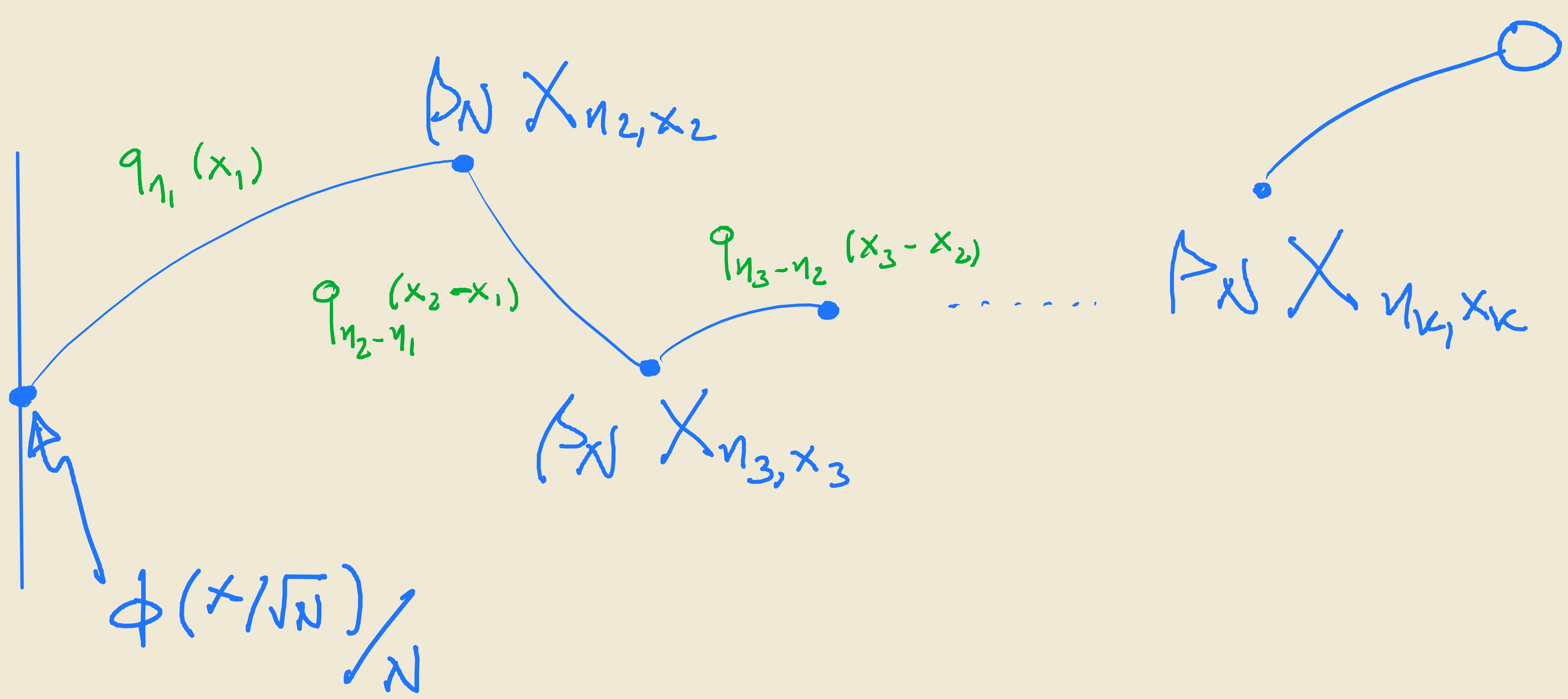
"Logarithmic Combinatorial Structures"

$$\rho(1/t) \simeq \text{Prob}\{\text{longest cycle in } S_N \le N t\}$$

Higher Moments

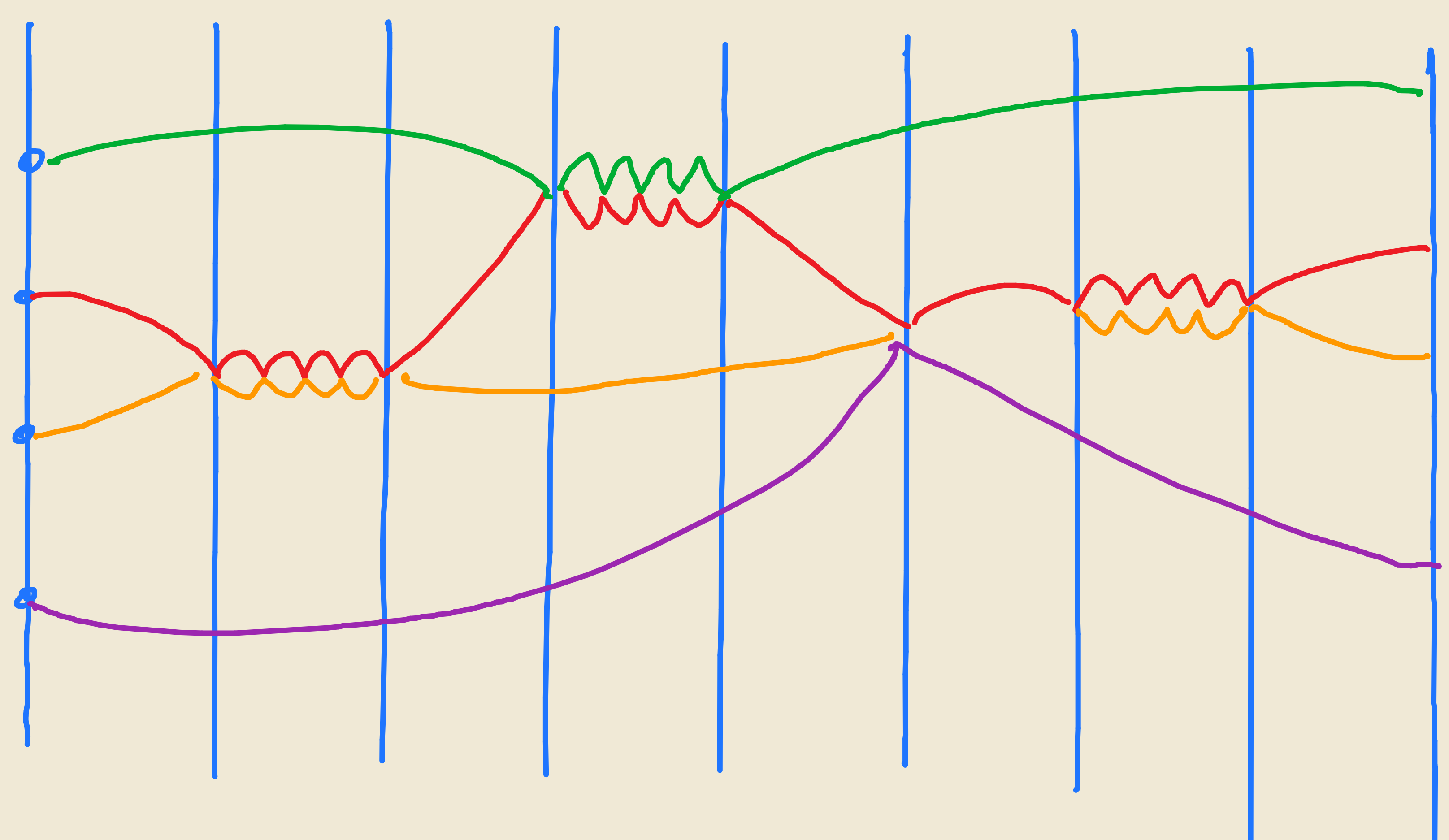
$$\mathbb{E}\left[\left(Z_N(Nt, \sqrt{N}x)\right)^4\right] =$$

$$= \mathbb{E}\left[\left(\sum_{k} \sum_{\substack{n_1 < \dots < n_k \\ x_1, \dots, x_k}}\right.\right.$$



$$\left.\left.\right)^4\right]$$

$$= \frac{1}{N^4} \prod_{i=1}^{4} \phi\left(\frac{x_i}{\sqrt{N}}\right) \sum_{k \geq 1} \sum_{n_1 < \dots < n_k}$$

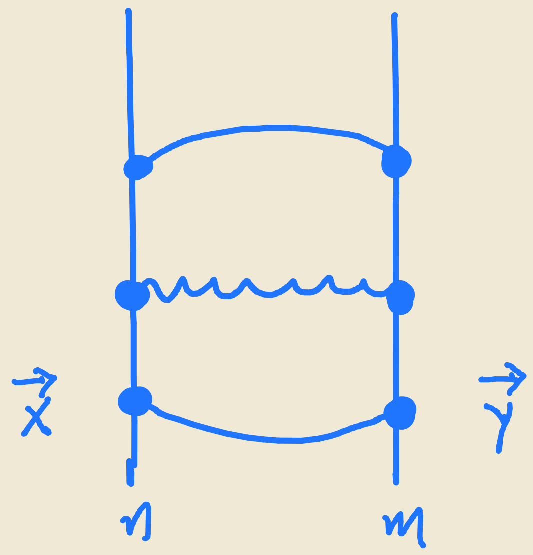


$n_1 \quad n_2 \quad n_3 \quad - - - - \quad n_k \quad N$

$$\cdot \prod_{i=1}^{4} \psi\left(\frac{z_i}{\sqrt{N}}\right)$$

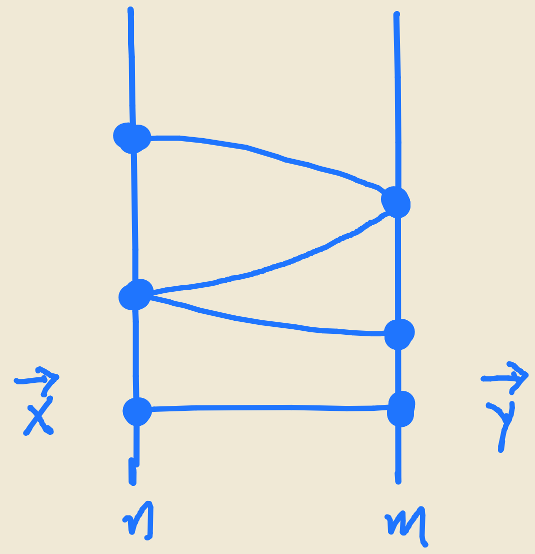
Operators

Dickman Evolution



$$\equiv U_{m-n}(\vec{x},\vec{y})$$

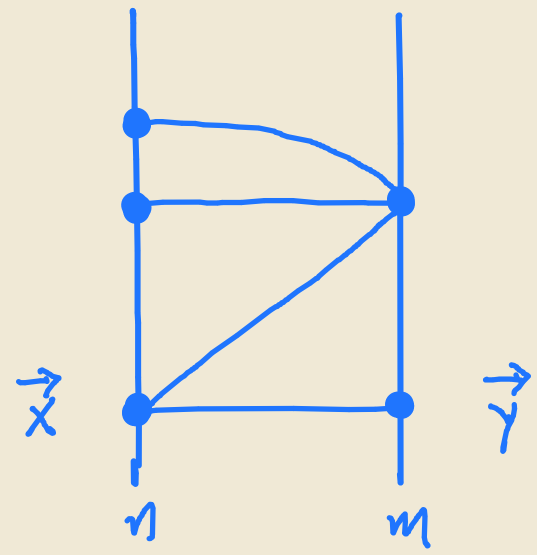
Free Evolutions



$$\equiv Q^{I,J}_{m-n}(\vec{x},\vec{y})$$

$I = \{1\}, \{2,3\}, \{4\}$

$J = \{1,2\}, \{3\}, \{4\}$



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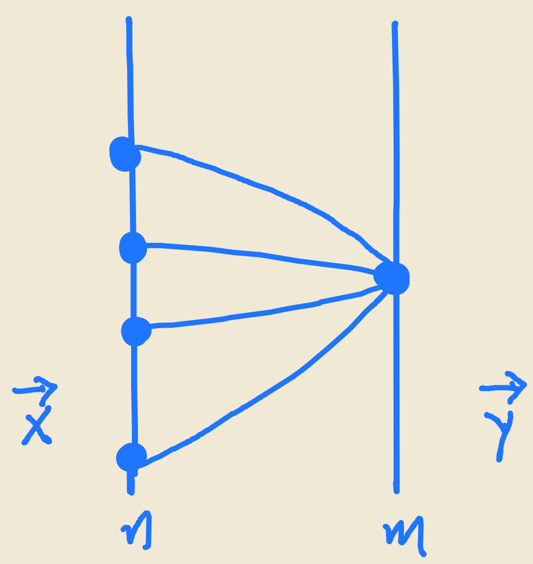
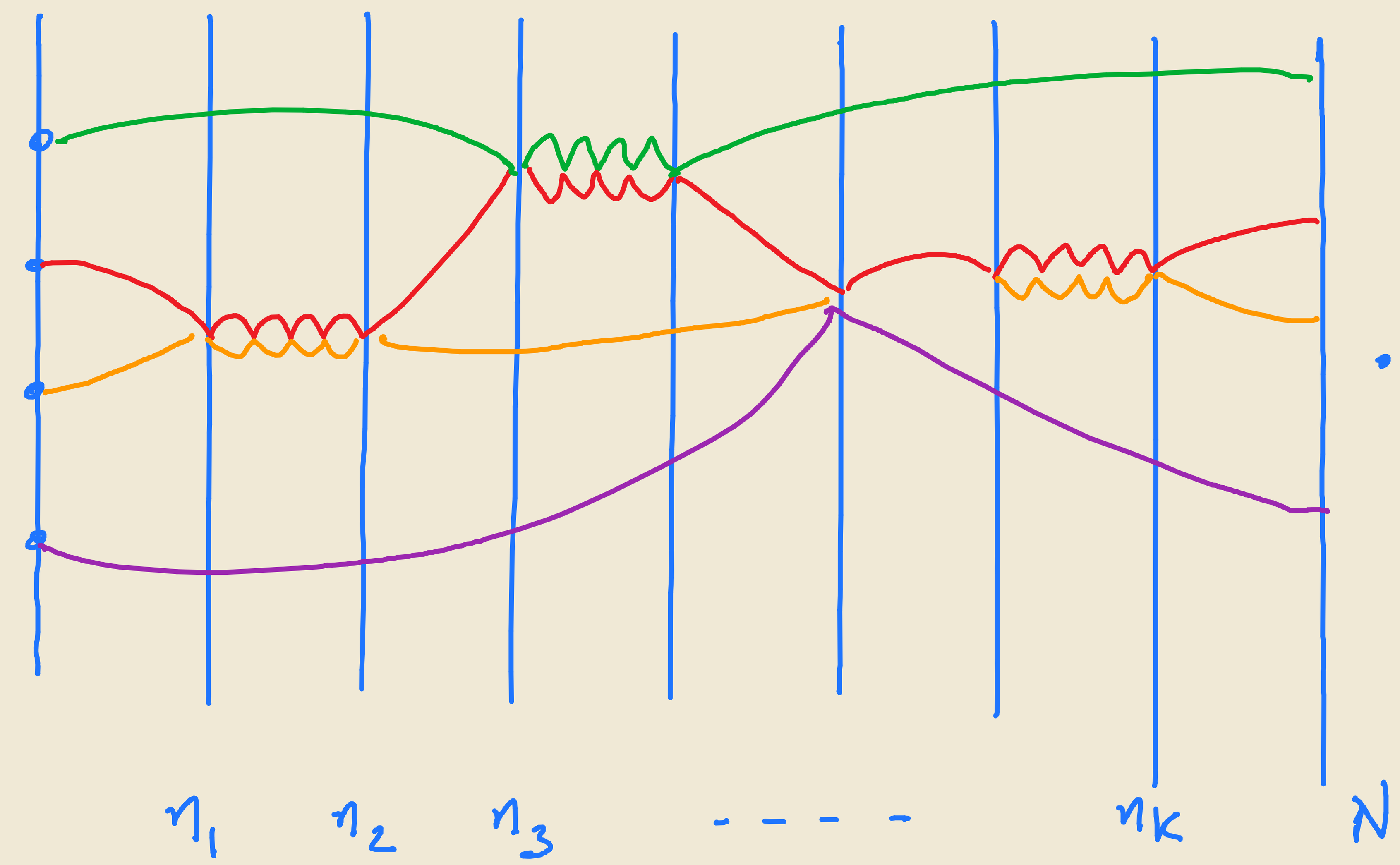


$$\equiv Q^{I,J}_{m-n}(\vec{x},\vec{y})$$

$I = \{1\}, \{2\}, \{3\}, \{4\} \equiv *$

$J = \{1,2,3,4\}$

$$\sum_{k\geq 1}\ \sum_{\substack{n_1<\cdots<n_k \\ \vec{x}_1,\dots,\vec{x}_k}} \frac{1}{N^4} \prod_{i=1}^{4} \phi\left(\frac{x_0^i}{\sqrt{N}}\right)$$



$$\cdot \prod_{i=1}^{4} \psi\left(\frac{z^i}{\sqrt{N}}\right)$$

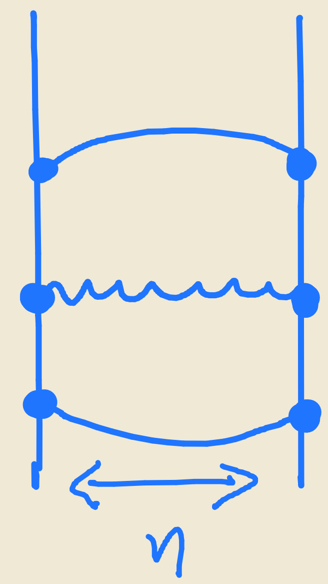
ignore multiple intersections

$$\rightsquigarrow \sum_{k\geq 1}\ \sum_{\substack{n_1<\cdots<n_k \\ \vec{x}_1,\dots,\vec{x}_k \\ I_1, I_2, \dots, I_k}} \frac{1}{N^4}\, \phi(\vec{x}_0)^{\otimes 4}\, Q^{*,I_1}_{n_1}(\vec{x}_0, \vec{x}_1) \prod_{i=1}^{k} U_{n_i - n_{i-1}}(\vec{x}_{i-1}, \vec{x}_i)\, Q^{I_i, I_{i+1}}_{n_{i+1}-n_i}(\vec{x}_i, \vec{x}_{i+1})\, \psi^{\otimes 4}(\vec{z})$$

Laplace Transforms

Dickman evolution

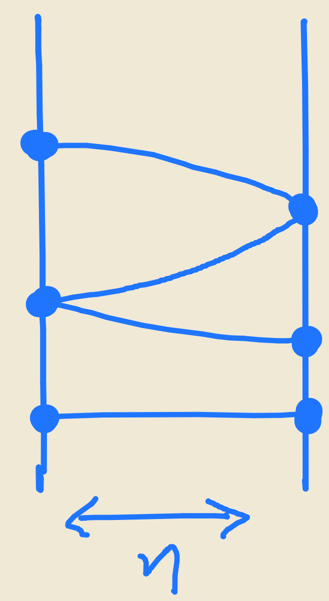
$$\sum_{n\geq 1} e^{-\lambda n/N} \quad \vec{x}$$



$$\vec{y} \equiv U_{\lambda,N}(\vec{x},\vec{y})$$

Free evolutions

$$\sum_{n\geq 1} e^{-\lambda n/N} \quad \vec{x}$$



$$\vec{y} \equiv Q^{I,J}_{\lambda,N}(\vec{x},\vec{y})$$

$$I = \{1\}, \{2,3\}, \{4\}$$
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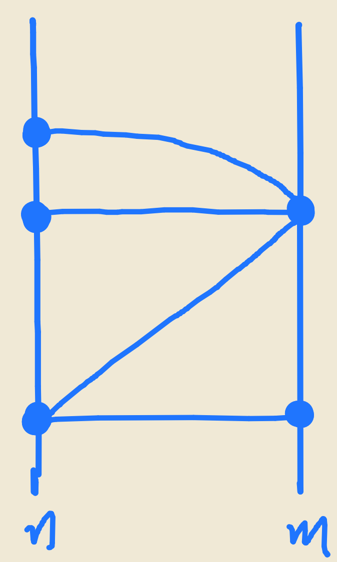
$$\sum_{n\geq 1} e^{-\lambda n/N} \quad \vec{x}$$



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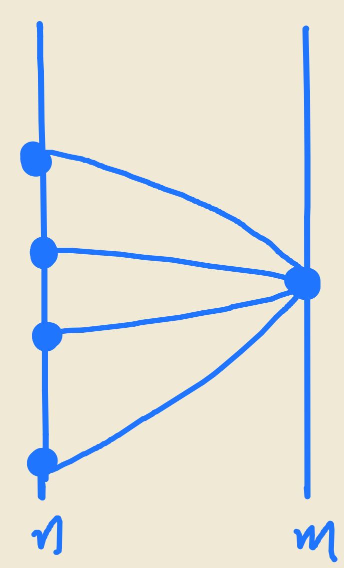
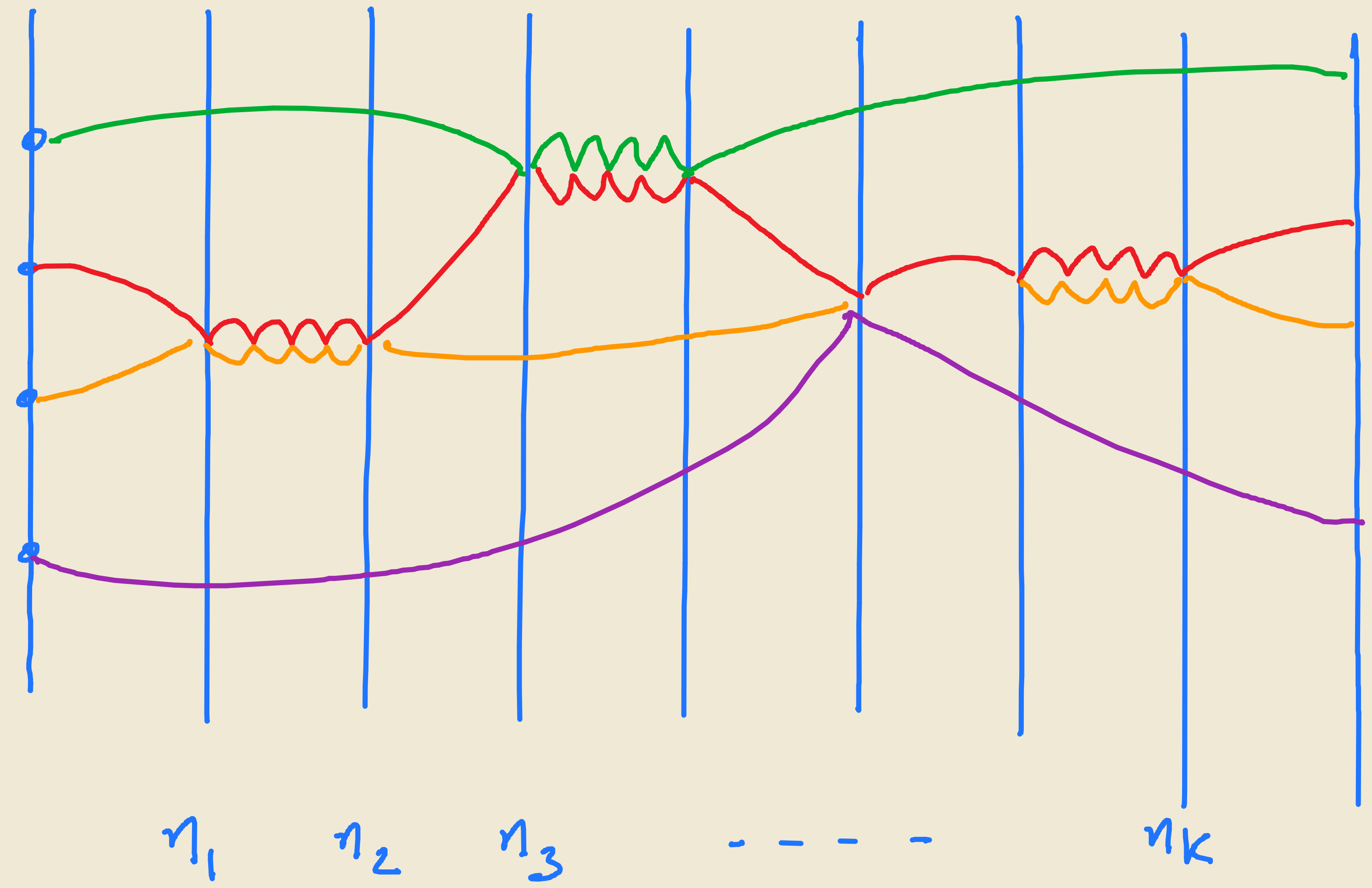


$$\vec{y} \equiv Q^{I,J}_{\lambda,N}(\vec{x},\vec{y})$$

$$I = \{1\}, \{2\}, \{3\}, \{4\} \equiv *$$
$$J = \{1,2,3,4\}$$

$$\sum_{k\geq 1}\ \sum_{\substack{n_1<\cdots<n_k \\ \vec{x}_1,\dots,\vec{x}_k}} \frac{1}{N^4}\prod_{i=1}^{4}\phi\left(\frac{x_0^i}{\sqrt{N}}\right)$$



$$\cdot \prod_{i=1}^{4}\psi\left(\frac{z^i}{\sqrt{N}}\right)$$

$$\leq e^{\lambda}\sum_{k}\sum_{I_1,\dots,I_k}\left\langle \frac{1}{N^4}\phi^{\otimes 4},\ Q_{\lambda,N}^{*,I_1}\ U_{\lambda,N}^{I_1}\ \cdots\ Q_{\lambda,N}^{I_{k-1},I_k}\ U_{\lambda,N}^{I_k}\ Q_{\lambda,N}^{I_k,*}\ \psi^{\otimes 4}\right\rangle$$

where $\langle f, Ag\rangle := \sum_{x,y} f(x)\,A(x,y)\,g(y)$

,

$$\leq e^{\lambda}\sum_{k}\ \sum_{I_1,\dots,I_k}\left\|\frac{1}{N^4}\phi^{\otimes 4}\right\|_p \left\|Q_{\lambda,N}^{*,I_1}\right\|_{q\to q}\left\|U_{\lambda,N}^{I_1}\right\|_{q\to q}\cdots$$

$$\cdots\left\|Q_{\lambda,N}^{I_{k-1},I_k}\right\|_{q\to q}\left\|U_{\lambda,N}^{I_k}\right\|_{q\to q}\left\|Q_{\lambda,N}^{I_k,*}\right\|_{q\to q}\left\|\psi^{\otimes 4}\right\|_q$$

Thm (CSZ)

for $f \in L^p((\mathbb{Z}^2)^3)$ & $g \in L^q((\mathbb{Z}^2)^3)$

$$\sum_{x,y \in (\mathbb{Z}^2)^3} f(x)\, U_{\lambda,N}(x,y)\, g(y) \leqslant \frac{C}{\log \lambda} \|f\|_p \|g\|_q$$

Proof Use Hölder & spatial extension of Dickman

$$U_{\lambda,N}(x,y) = \sum_{n=1}^{N} e^{-\lambda \frac{n}{N}} U_n(x,y)$$

$$\sim \sum_{n=1}^{N} e^{-\lambda_N} \frac{(\log N)^2}{N^2} G_\vartheta\left(\frac{n}{N}\right) g_{\frac{4n}{N}}\left(\frac{y-x}{\sqrt{N}}\right)$$

& recall that $G_\vartheta(t) \sim \dfrac{1}{t(\log t)^2}$ for $t \to 0$

Thm (CSZ) $x, y \in (\mathbb{Z}^2)^k$

$$Q^{I,J}_{\lambda,N}(x,y) := \delta_I(x) \cdot \sum_{n=1}^{N} e^{-\lambda \frac{n}{N}} \prod_{i=1}^{4} q_n(y^i - x^i) \cdot \delta_J(y)$$

then

$$\sum_{x,y} f(x)\, Q^{I,J}_{\lambda,N}(x,y)\, g(y) \leq C_{pq} \|f\|_p \|g\|_q$$

Remark (more general h-moment)

$$Q^{I,J}_{\lambda,N}(x,y) \lesssim \frac{\delta_I(x)\, \delta_J(y)}{|x-y|^{2h-2}}$$

$\rightsquigarrow$ Critical Hardy-Littlewood-Sobolev

Predecessor: Dell'Antonio - Figari - Teta '94

for $Q \rightsquigarrow$ Green's function of

$$-\Delta + \sum_{i<j} \delta(x_i - x_j)$$

Idea.

Hardy-Littlewood Sobolev ineq.

$$\sum_{x,y} f(x) \frac{1}{|x-y|^{\nu}} g(y) \overset{\text{Hölder}}{\leq} \left(\sum_{x,y} f(x)^p \frac{1}{|x-y|^{\nu}} \right)^{1/p} \cdot \left(\sum_{x,y} \frac{1}{|x-y|^{\nu}} g(y)^q \right)^{1/q}$$

$$\leq C \|f\|_p \cdot \|g\|_q$$

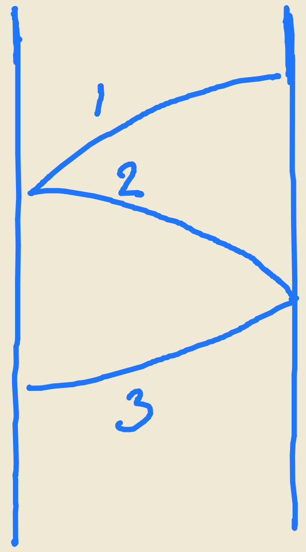
if $\sum_{y} \frac{1}{|x-y|^{\nu}} < \infty$ then ok

but Critical Hardy-Littlewood-Sobolev ineq.

$$\sum_{y \in \mathbb{Z}^d} \frac{1}{|x-y|^d} \sim \int \frac{dr}{r}$$

Trick

$$\left(\nu = 2h-2, \text{critical}\right)$$

$$\sum_{\substack{x_1 = x_2,\, x_3 \\ y_1,\, y_2 = y_3}}$$ 

$$= \sum_{\substack{x_1 = x_2,\, x_3 \\ y_1,\, y_2 = y_3}} f(x) \frac{1}{|x-y|^{\nu}} g(y)$$

$$= \sum_{\substack{x_1 = x_2,\, x_3 \\ y_1,\, y_2 = y_3}} f(x) \frac{1}{|x-y|^{\nu}} \frac{|x_2 - x_3|^{\alpha}}{|x_2 - x_3|^{\alpha}} \cdot \frac{|y_1 - y_2|^{\alpha}}{|y_1 - y_2|^{\alpha}} g(y)$$

$$\leq \left(\sum_{\substack{x_1 = x_2,\, x_3 \\ y_1,\, y_2 = y_3}} f(x)^{p} \frac{1}{|x-y|^{\nu}} \frac{|x_2 - x_3|^{\alpha p}}{|y_1 - y_2|^{\alpha p}} \cdot \right)^{1/p}$$

$$\cdot \left(\cdots\cdots \right)^{1/q}$$

Critical 2d SHF vs GMC

Gaussian Multiplicative Chaos

$$M_\gamma(dx) := e^{\gamma X(x) - \frac{\gamma^2}{2}\langle X \rangle} dx$$

with $X(\cdot)$ a Gaussian field, typically GFF

$$\mathbb{E}[X(x)X(y)] \sim \log|x-y|^{-1}, \quad \text{for } x-y \sim 0$$

Q. Is Critical 2d SHF = GMC ?

Moment Structures

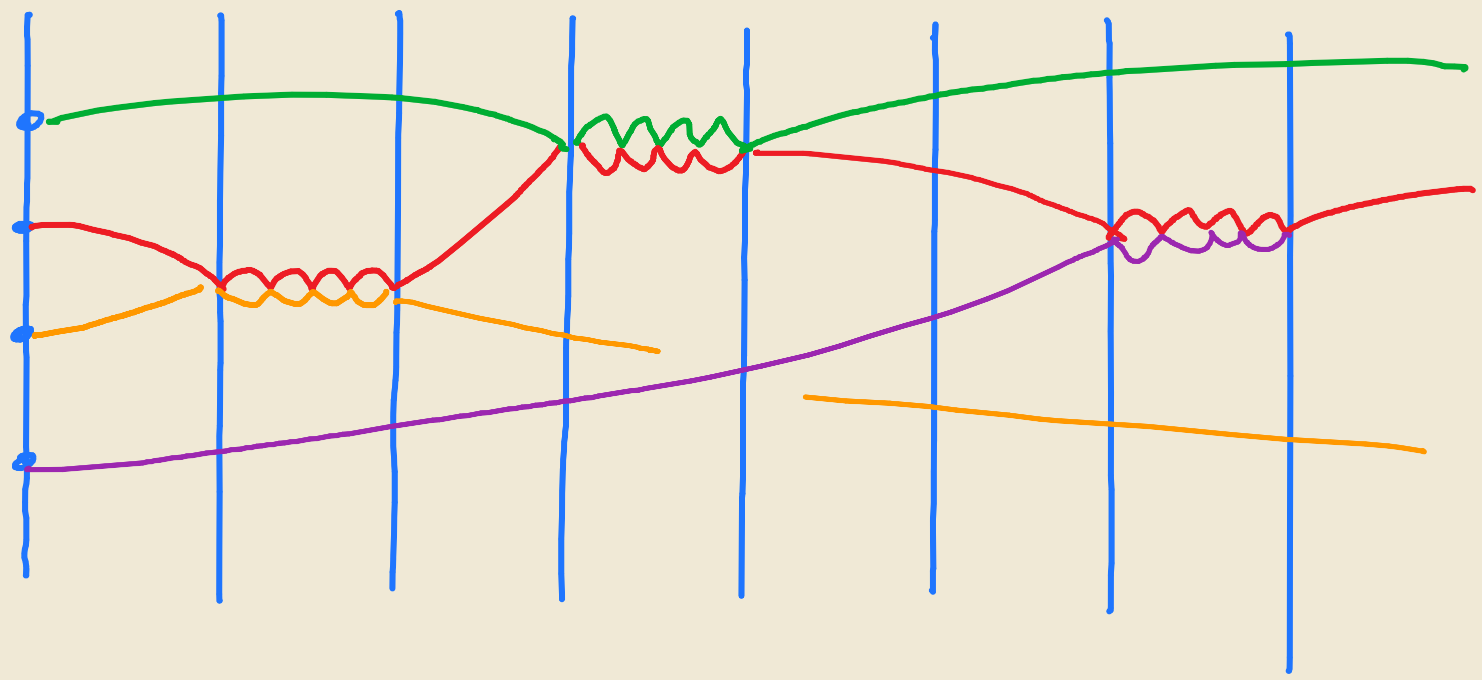
Critical 2d SHF

$$\mathbb{E}\left[\mathcal{Z}(dx)\,\mathcal{Z}(dy) \right] = K(x,y)\,dx\,dy$$

$$\text{with } K(x,y) \sim C \log |x-y|^{-1}$$

Mixed moments of Critical 2d SHF

$$\mathbb{E}\left[\mathcal{Z}(dx_1) \cdots \mathcal{Z}(dx_n) \right] =$$

$=$ 

Mixed moments of GMC

$$\mathbb{E}\left[M_\gamma(dx_1) \cdots M_\gamma(dx_n) \right] \overset{\text{Wick}}{=\!=} \prod_{1 \le i < j \le n} e^{K(x_i,x_j)} \, d\vec{x}$$

Thm (CSZ) for any nice, symmetric, non-increasing ϕ

$$\mathbb{E}\left[\left(\int_{\mathbb{R}^2} \phi(x)\, M_\gamma(dx)\right)^3\right] < \mathbb{E}\left[\left(\int_{\mathbb{R}^2} \phi(x)\, Z^{SHF}(dx)\right)^3\right]$$

Moments & Collisions of SRW

For $\beta_N = \hat{\beta} \frac{\sqrt{\pi}}{\sqrt{\log N}}$

$$\mathbb{E}\left[Z_{N,\beta_N}^h \right] = \mathrm{E}^{\otimes h}\left[e^{\beta_N \sum_{i<j} L_N^{ij}} \right]$$

Thm (Lygkonis-Z)

$$\left\{ \frac{\sqrt{\pi}}{\sqrt{\log N}} L_N^{ij} \right\}_{i<j\leq h} \xrightarrow{d} \left\{ \mathrm{Exp}^{ij}(1) \right\}_{i<j\leq h} \quad \text{i.i.d.}$$

Corollary for $\hat{\beta}<1$

$$\lim_{N\to\infty} \mathbb{E}\left[Z_{N,\beta_N}^h \right] = \lim_{N\to\infty} \mathrm{E}^{\otimes h}\left[e^{\beta_N \sum_{i<j} L_N^{ij}} \right]$$

$$= \left(\lim_{N\to\infty} \mathbb{E}\, Z_{N,\beta_N}^2 \right)^{\binom{h}{2}}$$

Thm (CSZ)

This fails at $\hat{\beta} = 1$ for the SHE / SHF

if J_ε is a Dirac approximation

$$\lim_{\delta \downarrow 0} \lim_{\varepsilon \downarrow 0} \frac{\int \cdots \int \prod_i g_\delta(x_i)\, E_{\vec{x}}^{\otimes h}\left[\prod_{i<j} e^{\frac{\sqrt{2\pi}}{|\log \varepsilon|} \int_0^t J_\varepsilon (B_s^i - B_s^j)\, ds} \right]}{\left(\int \int g_\delta(x_1)\, g_\delta(x_2)\, E_{\vec{x}}^{\otimes h}\left[e^{\frac{\sqrt{2\pi}}{|\log \varepsilon|} \int_0^t J_\varepsilon (B_s^i - B_s^j)\, ds} \right] \right)^{\binom{h}{2}}} > 1$$

Tool: Gaussian Correlation Inequality

first attempt to use GCI by Feng (PhD '16)

Some questions / directions

1. Axiomatic Characterisation of the Critical 2d SHF

2. Universality

3. Relations to GMC & other theories eg LQG

4. Fine properties of the measures
    - fractality
    - structure of maxima & thick points
    - regularity of the measure

5. flow properties & polymer measures / localisation

6. precise moment asymptotics

7. Renormalisation of 1-point

8. push towards Critical Theory of SPDEs

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4. Fine properties of the measures
   - fractality
   - structure of maxima & thick points
   - regularity of the measure

5. flow properties & polymer measures / localization

6. precise moment asymptotics

7. Renormalisation of 1-point

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The End